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PRICE RESPONSIVENESS AND MARKET CONDITIONS

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This paper develops a systematic relationship between the price responsiveness of an optimizing agent and the conditions prevailing in its relevant markets. The result rests on a formal comparative statics phenomenon that has a striking similarity to the classical strong LeChâtelier principle as well as to the interesting new comparative statics phenomenon established by Edlefsen.

IT IS WELL KNOWN that one can establish, under certain conditions, a systematic relationship between the intensity of the reactions of an optimizing agent to parameter changes, and the restrictiveness of the environment in which it operates. This phenomenon, the so-called strong[2] LeChâtelier principle, is formally merely a property of bordered Hessians, and a consequence of the fact that by subjecting an agent to additional 'just binding' constraints the curvature of the surface of its feasible set is being made progressively more concave at the chosen optimal point. Edlefsen **[3]** has recently shown that essentially the same predictions as under the LeChâtelier principle follow if the increased concavity of the feasible set's curvature at the extremum point is brought about by the replacement of a given constraint by another more concave one rather than by addition of further constraints. This type of problem may appear to be somewhat artificial at first sight; but it arises naturally in the context of hedonic price functions, and as Edlefsen demonstrated, its analysis leads to new and very interesting insights into the effects of nonlinearities in the constraints on certain aspects in the behavior of households, for example.

To see what type of predictions emerge from Edlefsen's approach consider Figure 1 which depicts the decision problem of two agents maximizing revenues subject to the production possibility frontier $g^1(x_1, x_2) = 0$ and $g^2(x_1, x_2) = 0$, respectively. Clearly, if prices are such that $f(x_1, x_2) = c$ is the relevant isorevenue line, then both agents will choose the same point (x_1^*, x_2^*). Now, change one price slightly so that the isorevenue line turns a bit. It is then obvious that the decisions of the two agents will diverge. But what is equally obvious and more important in this context is that the quantitative response to this price change will be comparatively less pronounced for the agent which has to observe the more strongly curved frontier g^2. This simple example demonstrates why and how the curvature of the feasible set at the chosen optimal point influences the intensity of certain comparative static reactions.

As has already been indicated, and is also apparent from the type of proof Edlefsen uses, his approach is still very much in the LeChâtelier tradition: the

[1] I wish to thank the editors of *Econometrica* and two anonymous referees for very helpful suggestions.

[2] The helpful distinction between the strong and the so-called weak LeChâtelier principle is due to Eichhorn and Oettli **[4]**. The basic reference is, of course, Samuelson **[11]**. For a more recent treatment, see Silberberg **[12]**, Kusumoto **[8]**, Fujimoto **[6]**, and Hatta **[7]**.

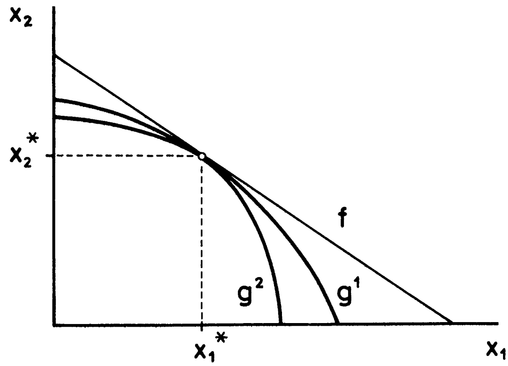

FIGURE 1.

feasible set is varied systematically whereas the objective function is held fixed. One may wonder whether the reverse, i.e., varying the objective function systematically while keeping the feasible set fixed, would not lead to a similar phenomenon—and indeed, it does. It is the purpose of this paper to briefly develop this idea. Section 1 sets out with a short description of the type of static optimization problems which are to be compared and goes on to prove the main result in two versions. In Section 2, the stronger version will then be applied to a simple problem in the theory of the firm. Our aim there is to establish a systematic relationship between the price responsiveness of a producer and the conditions prevailing in its markets.

1. THE EFFECTS OF NONLINEARITIES IN THE OBJECTIVE FUNCTION ON THE BORDERED HESSIAN

In what follows, we compare the comparative statics of the two optimization problems

$$(1) \qquad \max_x f(x,\alpha) \qquad \text{subject to} \quad g(x,\alpha)=0$$

and

$$(1') \qquad \max_x \hat{f}(x,\alpha) \qquad \text{subject to} \quad g(x,\alpha)=0,$$

which differ only in that the scalar valued objective functions $f(x,\alpha)$ and $\hat{f}(x,\alpha)$ may differ. The vector of decision variables, $x=(x_1,\ldots,x_n)'$, the vector of parameters, $\alpha=(\alpha_1,\ldots,\alpha_m)'$, and the vector valued constraint function, $g=(g^1,\ldots,g^r)'$, are the same in both problems. Define the associated Lagrangeans

$$(2) \qquad L(x,\lambda,\alpha)=f(x,\alpha)-\lambda' g(x,\alpha)$$

and

$$(2') \qquad \hat{L}(x,\hat{\lambda},\alpha)=\hat{f}(x,\alpha)-\hat{\lambda}' g(x,\alpha),$$

where $\lambda = (\lambda_1, \ldots, \lambda_r)'$ and $\hat{\lambda} = (\hat{\lambda}_1, \ldots, \hat{\lambda}_r)'$ denote the respective Lagrange multipliers.

Throughout it is assumed that the two problems achieve a regular interior maximum for any given set of parameters, i.e., we assume that at these extremum points x^* and $\hat{x}^*$, the respective first and second order sufficient conditions hold. This allows us to conclude immediately that the associated bordered Hessians

$$(3) \qquad H(x,\lambda,\alpha) = \begin{bmatrix} f_{xx} - \lambda' g_{xx} & -g_x' \\ -g_x & 0 \end{bmatrix}$$

and

$$(3') \qquad \hat{H}(x,\hat{\lambda},\alpha) = \begin{bmatrix} \hat{f}_{xx} - \hat{\lambda}' g_{xx} & -g_x' \\ -g_x & 0 \end{bmatrix}$$

are regular at the points (x^*,λ^*,α) and $(\hat{x}^*,\hat{\lambda}^*,\alpha)$, respectively, and that the submatrices A and $\hat{A}$ of their conformably partitioned inverses

$$(4) \qquad H^{-1}(x^*,\lambda^*,\alpha) = \begin{bmatrix} A & B' \\ B & C \end{bmatrix}$$

and

$$(4') \qquad \hat{H}^{-1}(\hat{x}^*,\hat{\lambda}^*,\alpha) = \begin{bmatrix} \hat{A} & \hat{B}' \\ \hat{B} & \hat{C} \end{bmatrix}$$

are negative semidefinite.[3] We state the structure of these bordered Hessians and their inverses rather explicitly since the result below is expressed in terms of the relationship between these inverse bordered Hessians which play, as is well known, a crucial rôle in the comparative static analysis.

Bordered Hessians have not been popular because most results in the comparative statics of optimization problems can be obtained by more powerful and direct methods. For instance, in the conjugate pairs case the typical signing of the response to a conjugate parameter can be done most easily by using the weak LeChâtelier principle. And even more complex results such as reciprocity relations, the negative semidefiniteness of the substitution matrix, or even statements under the strong LeChâtelier principle regarding the intensity of responses in a more or less restricted environment follow almost trivially when the gain function approach is used.[4] However, these more direct methods appear to break down when the conjugate pairs property is not available or—as in our case—when there is no guarantee that the target functions which reflect a particular environment assume identical values at the initial optimal point. We, therefore, have little choice but to proceed along the traditional, more indirect route, and investigate bordered Hessians.

[3] See Debreu [**2**] or Quirk [**10**, pp. 23–24] who gives a simple proof.
[4] For examples see Silberberg [**12**] or Hatta [**7**].

So far, we have not specified in what sense the objective function $\hat{f}(x,\alpha)$ is to differ from the objective function $f(x,\alpha)$. In essence, we assume that a level curve of the objective function $\hat{f}(x,\alpha)$ is tangent to the level curve of the objective function $f(x,\alpha)$ at the point x^*, but bends away from this tangent hyperplane more quickly than the latter. More precisely, we investigate the effect of letting $\hat{f}(x,\alpha)$ possess the same gradient as $f(x,\alpha)$ at the point x^*, but be more concave. This is the reverse analogue to the case covered by Edlefsen or the classical strong LeChâtelier principle. The result then is: (i) if the comparability condition

$$\hat{f}_x(x^*,\alpha) = f_x(x^*,\alpha) \tag{5}$$

holds, problems (1) and (1′) lead to the same decision, i.e., $\hat{x}^* = x^*$. Furthermore, $\hat{\lambda}^* = \lambda^*$. (ii) Given the comparability condition (5), if

$$h'(\hat{f}_{xx}(x^*,\alpha) - f_{xx}(x^*,\alpha))h \leqq 0 \qquad \text{for all} \quad h = (h_1, \ldots, h_n)' \tag{6}$$

holds, the difference $H^{-1}(x^*,\lambda^*,\alpha) - \hat{H}^{-1}(\hat{x}^*,\hat{\lambda}^*,\alpha)$ will be negative semidefinite. (iii) Given the comparability condition (5), if the weaker condition

$$h'(\hat{f}_{xx}(x^*,\alpha) - f_{xx}(x^*,\alpha))h \leqq 0 \qquad \text{for all } h \text{ such that} \quad g_x(x^*,\alpha)h = 0 \tag{7}$$

holds, then still the difference $A - \hat{A}$ will be negative semidefinite.

PROOF: Compare the first order conditions associated with (1) and (1′), which are

$$f_x(x^*,\alpha) - \lambda^{*\prime} g_x(x^*,\alpha) = 0, \qquad g(x^*,\alpha) = 0, \tag{8}$$

and

$$\hat{f}_x(\hat{x}^*,\alpha) - \hat{\lambda}^{*\prime} g_x(\hat{x}^*,\alpha) = 0, \qquad g(\hat{x}^*,\alpha) = 0, \tag{8′}$$

respectively, and observe that any solution (x^*,λ^*) solving (8) solves (in view of (5)) likewise (8′). Thus we conclude that $\hat{x}^* = x^*$ and $\hat{\lambda}^* = \lambda^*$, proving (i). In particular, the bordered Hessians and their inverses are to be evaluated at the same point (x^*,λ^*,α). Note that $\hat{H}(\hat{x}^*,\hat{\lambda}^*,\alpha)$ can accordingly be expressed as

$$\hat{H}(\hat{x}^*,\hat{\lambda}^*,\alpha) = H(x^*,\lambda^*,\alpha) + S, \tag{9}$$

where S has the simple structure

$$S = \begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix} \qquad \text{with} \quad D = \hat{f}_{xx}(x^*,\alpha) - f_{xx}(x^*,\alpha). \tag{10}$$

Using (9), we have $\hat{H}H^{-1}\hat{H} = H + 2S + SH^{-1}S$ and $\hat{H}\hat{H}^{-1}\hat{H} = H + S$. Subtracting these two equations, and multiplying the result from both sides by $\hat{H}^{-1}$, gives $H^{-1} - \hat{H}^{-1} = \hat{H}^{-1}S\hat{H}^{-1} + \hat{H}^{-1}SH^{-1}S\hat{H}^{-1}$. Set $v' = (u', w')$, where u and w are n- and r-vectors, respectively, and multiply the last equation from both sides with this vector. Multiplying through and using (4), (4′), and (10) gives

$$(11)\qquad v'(H^{-1}-\hat{H}^{-1})v=(u'\hat{A}+w'\hat{B})D(\hat{A}u+\hat{B}'w) + ([u'\hat{A}+w'\hat{B}]D)A(D[\hat{A}u+\hat{B}'w]).$$

Now, the second term on the right-hand side is clearly nonpositive, since A must be negative semidefinite by the second order sufficient conditions. If also D is unconditionally negative semidefinite, then the entire right-hand side is nonpositive. Thus, in view of (6), (ii) is proven. Let $w = 0$ and reconsider (11), which, using (4) and (4′) again, becomes

$$(12)\qquad u'(A-\hat{A})u=(u'\hat{A})D(\hat{A}u)+(u'\hat{A}D)A(D\hat{A}u).$$

The second term on the right-hand side is clearly nonpositive. In order to establish the nonpositivity of the entire right-hand side of (12) and thus to prove (iii), we must show that the n-vector $h=\hat{A}u$ occurring in the first term on the right-hand side has the property that is required in (7). Observe that, when multiplying out the equation $\hat{H}\hat{H}^{-1}=I$ in partitioned form using (3′) and (4′), one of the resulting four equations is $g_x\hat{A}=0$. Hence $g_x\hat{A}u=0$ holds, implying that the vector $h=\hat{A}u$ indeed possesses the required property $g_xh=0$. This establishes (iii) and completes the proof.

Condition (7) is much weaker than condition (6); for it requires the relative concavity of $\hat{f}(x,\alpha)$ to $f(x,\alpha)$ to hold only for those directions around x^* that are feasible given the constraints, while (6) requires this to hold for all directions. However, unless the problem under consideration is of an exceedingly simple structure, the validity of condition (7) will in general be rather difficult to check if (6) is violated.

2. AN APPLICATION TO THE THEORY OF THE FIRM

Edlefsen's main result on the systematic change in the substitution matrix of a household when facing parametric or hedonic prices [**3**, Theorem 3.1] can be deduced by considering a household that minimizes expenditures, given a certain prescribed utility level, and using (iii) above. On this basis one could likewise show that the compensated demand of a household entitled to progressive rebates on its consumer goods purchases is more sensitive to a change in a conjugate parameter, such as an indirect tax or a subsidy on a commodity, than a household without rebates.

In order to investigate a problem which cannot be solved that easily with Edlefsen's approach we analyze here in more detail the comparative static reactions of two producers who have identical technological and other constraints, but face different market conditions. It is a well known fact that, e.g., a profit maximizing monopolist reacts qualitatively much the same way to a price change in a competitive factor market as a producer facing competitive conditions in his output market. But while the direction of these reactions to such price changes are identical, the intensity in general differs. As a matter of fact, it can be shown that, under fairly reasonable assumptions, the monopolist responds less intensely to a price change in a factor market than a perfect competitor. One might be inclined to think that it is essentially the elasticity of demand or supply in the various relevant output or input markets which determines the intensity of the response. This is not correct: it is the speed with which marginal revenue or marginal expenditure in the various markets changes, rather than the size of the demand or supply elasticities in these markets, which is responsible for a more or less intense reaction to a change in a conjugate parameter. To put it more precisely and generally, we intend to show that the more rapidly marginal revenue declines or marginal expenditure rises in at least one output or input market, the less intensely a profit maximizing producer tends to react in any market to a change in prices, indirect taxes, shifts in demand or supply, or any other conjugate parameter. This finding allows us to portray the general demand and supply behavior of a producer, with monopolistic or monopsonistic power in a certain market j, as being bracketed between two extremes, namely behavior under perfect competition, and behavior under quantity rationing in market j.

Letting $x = (x_1, \ldots, x_n)'$ denote quantities, we use the common convention that outputs are positive, inputs negative. Let p_i, the (positive) price prevailing in market i, be a function of x_i alone[5] and define $z^i(x_i) = x_i p_i(x_i)$ as the gross revenue or, if negative, the gross expenditure in market i. Consider then a producer solving the profit maximization problem

$$(13) \qquad \max_x \beta' z(x) \qquad \text{subject to} \quad h(x) - \gamma \leqq 0,$$

where $z(x) = (z^1(x_1), \ldots, z^n(x_n))'$ is the vector of gross revenues and expenditures, $\beta = (\beta_1, \ldots, \beta_n)'$ is a vector of strictly positive shift parameters (which may accommodate taxes, subsidies, exchange rates etc.), such that the product $\beta' z(x)$ is net profits.

Writing the constraints in the inequality form $h(x) - \gamma \leqq 0$ where $h(x) = (h^1(x), \ldots, h^r(x))'$ and $\gamma = (\gamma_1, \ldots, \gamma_r)'$, one may generally interpret $h^j(x)$ as the use of resource j, which must not exceed the available amount γ_j. More specifically, if the kth constraint is a production function, the associated γ_k can be viewed as an additive technological shift parameter. We will assume, incidentally, that all inactive constraints have already been deleted so that at the optimum we have $h(x^*) - \gamma = 0$.

[5] Letting the prices p_i be functions of x_i alone rather than of the entire vector x precludes the possibility of allowing for interconnected markets and appears therefore to be unwarranted. However, dropping this assumption would substantially complicate the comparative static analysis below and lead to results that appear to resist a useful economic interpretation. This will become more apparent when viewing (14), (15), and (16) below and considering that without this assumption the relevant matrices z_x and $\beta' z_{xx}$ would cease to be diagonal.

Writing $L = \beta' z(x) - \lambda'(h(x) - \gamma)$ for the associated Lagrangean, the complete comparative static system for this producer is given by

$$\begin{bmatrix} x_\beta^* & x_\gamma^* \\ \lambda_\beta^* & \lambda_\gamma^* \end{bmatrix} = -H^{-1}(x^*, \lambda^*, \beta, \gamma) \begin{bmatrix} z_x(x^*) & 0 \\ 0 & I \end{bmatrix}.$$

Using the partitioning proposed in (4), the more promising parts of these comparative static reactions are

(14.1) $x_\beta^* = -Az(x^*)$,

(14.2) $\lambda_\gamma^* = -C$.

Consider now a second producer solving

(13′) $\max_x \beta' \hat{z}(x)$ subject to $h(x) - \gamma \leqq 0$,

and the associated comparative static system, the relevant parts of which, in obvious notation, are

(14.1′) $\hat{x}_\beta^* = -\hat{A}\hat{z}_x(\hat{x}^*)$,

(14.2′) $\hat{\lambda}_\gamma^* = -\hat{C}$.

This second producer faces the same constraints as the first producer, but is exposed to different market conditions, reflected by a different profit function, or more precisely, by different gross revenue and expenditure functions.

Assume now that marginal profits are identical at the point x^* so that

(15) $\beta' \hat{z}_x(x^*) = \beta' z_x(x^*)$,

and that the difference between the two profit functions is simply that the profit function of the second producer is more concave than that of the first producer at this point, i.e.

(16) $h'(\beta' \hat{z}_{xx}(x^*) - \beta' z_{xx}(x^*))h \leqq 0$ for all h.

Since z_x and likewise $\hat{z}_x$ are diagonal by assumption, (15) is tantamount to requiring $\partial \hat{z}^i(x_i^*)/\partial x_i = \partial z^i(x_i^*)/\partial x_i$ for $i = 1, \ldots, n$. Therefore marginal (gross) revenues and expenditures in all markets have to be identical for both producers at the point x^*. Since also $\beta' \hat{z}_{xx}$ and $\beta' z_{xx}$ are diagonal matrices, (16) requires, in effect, $\partial^2 \hat{z}^i(x_i^*)/\partial x_i^2 \leqq \partial^2 z^i(x_i^*)/\partial x_i^2$ for all $i = 1, \ldots, n$. Thus, for the second producer, in any market marginal gross revenue has to fall or marginal expenditure rise at least as rapidly as for the first producer.

Now, (15) is nothing but the comparability condition (5). Hence, by (i),

(17) $\hat{x}^*(\beta, \gamma) = x^*(\beta, \gamma)$ and $\hat{\lambda}^*(\beta, \gamma) = \lambda^*(\beta, \gamma)$.

This says that both producers make the same decision and face identical shadow prices. Since (16) is in turn equivalent to condition (6), we conclude furthermore on the basis of (ii) above that the difference $H^{-1}(x^*, \lambda^*, \beta, \gamma) - \hat{H}^{-1}(\hat{x}^*, \hat{\lambda}^*, \beta, \gamma)$ is negative semidefinite. Consequently

(18) $A - \hat{A}$ and $C - \hat{C}$ are negative semidefinite.

Using (14), (14′), (15), and (17) we have $\hat{x}^*_\beta - x^*_\beta = (A - \hat{A})z_x(x^*)$ and $\hat{\lambda}^*_\gamma - \lambda^*_\gamma = C - \hat{C}$. In view of (18), these imply in particular[6]

$$(19.1) \qquad \left|\frac{\partial \hat{x}_i^*}{\partial \beta_i}\right| \leqq \left|\frac{\partial x_i^*}{\partial \beta_i}\right| \qquad (i = 1, \ldots, n)$$

and

$$(19.2) \qquad \frac{\partial \hat{\lambda}_j^*}{\partial \gamma_j} \leqq \frac{\partial \lambda_j^*}{\partial \gamma_j} \qquad (j = 1, \ldots, r).$$

(19.1) shows that, the steeper the decline in marginal revenue or the rise in marginal expenditure in at least one market, the less intense is the reaction of a profit maximizing producer in any market to a change in a conjugate parameter. This phenomenon is entirely plausible. Consider for example the simple case of a producer facing competitive conditions in all markets and compare his behavior with that of a producer in exactly the same situation except that marginal expenditure in market j does not stay constant but rises with increasing demand. He is thus a monopsonist in this factor market. Let the price of an output i rise. For both producers, this provides an incentive to expand production of output i. But the incentive for the second producer erodes more quickly, since in the process of expanding output, the use of factor j normally[7] also expands and thus becomes more expensive, while for the first producer its price stays the same.

Let us stick to this simple setup one more moment. When starting from perfectly competitive conditions in factor market j and allowing these market conditions to become increasingly more 'monopsonistic' so that the second derivative of the expenditure function $z^j(x_j)$ goes from first zero to (minus) infinity, one may establish on the basis of (ii) a chain of increasingly weaker reactions $\partial x_i^*/\partial \beta_i$ $(i = 1, \ldots, n)$ the end of which simulates the situation of quantity rationing in factor market j. It is in this sense that the demand and supply behavior of an ordinary monopsonist in factor market j or, more precisely, his immediate reactions $\partial x_i^*/\partial \beta_i$ $(i = 1, \ldots, n)$, can be considered to be bracketed between those of a comparable producer facing perfect competition in

[6] Note that $\partial x_i^*/\partial \beta_i = -a_{ii}\, \partial z^i(x_i^*)/\partial x_i$ where a_{ii} denotes the ith diagonal element of A which must be negative semidefinite. The reaction $\partial x_i^*/\partial \beta_i$ has therefore the same sign as $\partial z^i/\partial x_i$. Since that applies also for the reaction $\partial \hat{x}_i^*/\partial \beta_i$, both reactions will consequently have the same sign and differ merely in magnitude.

[7] Alternatively, let factor j be inferior with respect to output i in the sense that demand for that factor falls when the price of output i rises (compare Bear [1] or Ferguson [5] for a symmetric but slightly different definition of an inferior input). In this case the marginal expenditure saved because of the reduction in demand for that factor is absolutely smaller for the second producer which again leads to the conclusion that his output expansion will be comparatively less pronounced.

market j on the one hand and those of a producer facing quantity rationing in this market on the other.

(19.2) also has an interesting interpretation. In view of the inequality constraints considered here, the Lagrange multipliers must be nonnegative. One would furthermore expect that these shadow prices λ_j^* and the allocations of the respective resources γ_j tend to move in opposite directions. Indeed, it can be shown that, under fairly general conditions, $\partial\lambda_j^*/\partial\gamma_j \leqq 0$ $(j = 1, \ldots, r)$.[8] With this case in mind, and interpreting the shadow prices as a measure of the incentive to overcome a given resource constraint by a marginal step, (19.2) says that the quicker marginal revenue in a product market falls or marginal expenditure in a factor market rises, the more rapidly does the incentive to overcome any resource bottleneck fade away. Or, if γ_j has the interpretation of a technological shift parameter, then (19.2) says that a monopolist's or a monopsonist's willingness to pay for a technological shift declines more rapidly than that of a comparable perfect competitor and less rapidly than that of a producer rationed in the respective market. (19.2) thus points at just another way in which monopolistic or monopsonistic power (and even more so quantity rationing in any market) tends to dampen the level of activity of a producer.

3. SUMMARY

Edlefsen [3] has shown that a property very similar to the strong LeChâtelier principle can be established when altering the feasible set of an optimizing agent by suitably replacing existing constraints rather than adding new ones. Here it was demonstrated that essentially the same phenomenon occurs when altering the objective function in a systematic manner rather than the feasible set. The result obtained is general enough to imply Edlefsen's result on the substitution matrix for a household choosing between quantity and quality when facing hedonic prices. We have also applied it to the theory of the firm: it was demonstrated that a systematic relationship exists between the intensity of the reactions of a producer to changes in a conjugate parameter and the conditions prevailing in his markets. To put the conclusion more pointedly, our analysis suggests that one may view behavior under monopoly or monopsony as bracketed by behavior under competition on the one side and behavior under rationing on the other.

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[8] See Leblanc and Van Moeseke [9, Proposition 1].

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